

# Optimum Population Size

Not too big, not too small,  
but just right

Econ/Demog c175

Prof. Josh Goldstein

Week 2, Lecture A

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# Our agenda

- What do we mean by the optimum?
- Two specific examples from Sauvy:
  - **economic** optimum
  - **power** optimum
- Also:
  - **minimum** population
  - **maximum** population
- Ordering the optima by population size
- Population “evolution”

# A happy medium?

“If there is such a thing as over-population and such a thing as under population, it follows that between the two there must be such a thing as just the right population”

-- H.P. Fairchild

# What size population is best?

## A slippery question

- Who is asking it?
  - A peasant, a worker, the average citizen?
  - A king, a capitalist, a “society”?
- What do they want to maximize?
  - average welfare
  - longevity
  - cultural power
  - military power
  - sum of all welfare

# Our approach

- Multiple answers  $\rightarrow$  optima (not optimum)
- Our goal: a broadly applicable framework to help us think about optimum population
- We focus on two specific examples
  - (1) Economic optimum
  - (2) Power optimum
- Introduce concepts (MP, TP, ...) that we'll see again.

# Contradictions

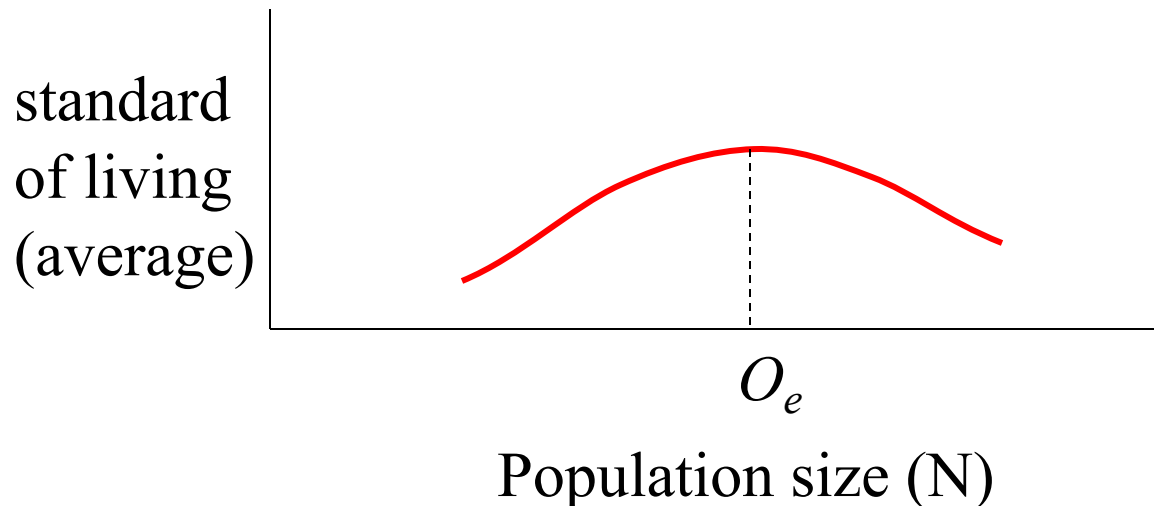
- Savvy points to failure of understanding of those who define the optimum as when "the population arrived at its greatest **strength** and **comfort**"
- Bentham, the utilitarian, phrase: "the **greatest amount of good** for the **greatest number**"
- There's usually a trade-off

# “Economic” optimum (issues)

- Many possible economic objective functions
- Sauvy’s “economic optimum” maximizes **average standard of living**
- Note:
  - We assume we can define “standard of living”
  - An individual perspective: size benefits not “society” but rather the individual via his or her standard of living
  - Inequality not part of the objective function

# Where is economic optimum?

- We assume fixed technology and resources
- Under-population = can't exploit the technology
- Over-population = run into resource constraints
- *Example: what is optimum size for a Freshman English seminar? (a) 1, (b) 2-4, (c) 5-9, (d) 10+*





# Another example: family size

- How many people should a "marriage" have in order to maximize welfare of spouses?

# Another example: family size

- How many people should a "marriage" have in order to maximize welfare of spouses?
- How many children should parents have (assume, for now, altruism, that they derive utility from their children's wellbeing)

# Some reasons for being "too small"

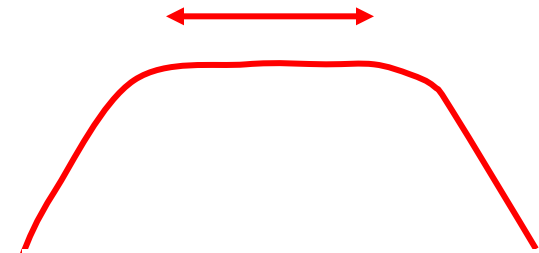
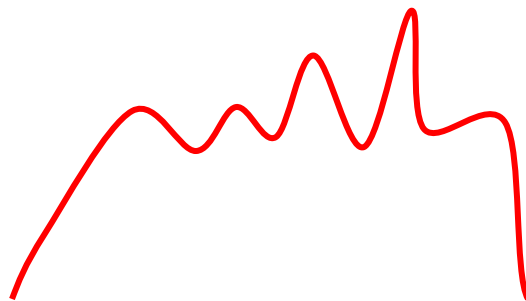
- Can't *specialize* (Smith on pins)  
([https://www.econlib.org/library/Smith/smWN.html?chapter\\_num=4#book-reader](https://www.econlib.org/library/Smith/smWN.html?chapter_num=4#book-reader))
- Can't *complement* (sex, holding a ladder to change a bulb)
- Can't afford *fixed cost investments* (roads, a machine, a teacher, ..)
- What else?

# Reasons for being "too big"

- "diminishing marginal returns"
- Classical version: variability in quality of inputs (so we choose best acre of land 1<sup>st</sup>, and it gets worse from there)
- Neo-classical version: with fixed capital, beyond some point, each additional worker will be less productive

# Critiques of the idea of optimum population size

- (1) We assumed fixed technology; but maybe each density has its own appropriate technology (later we'll read Boserup)
- (2) Perhaps maximum is not a single point -- rather, a wide range



# An example:

N	1	2	3	4	5	6
MP	0	4	3	2	1	0
AP	1	2				

*Copy on a separate sheet (also for later)*

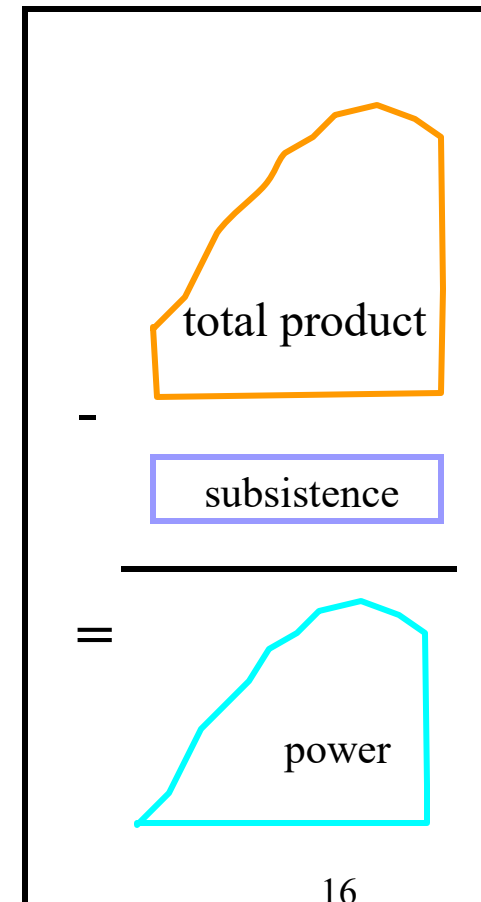
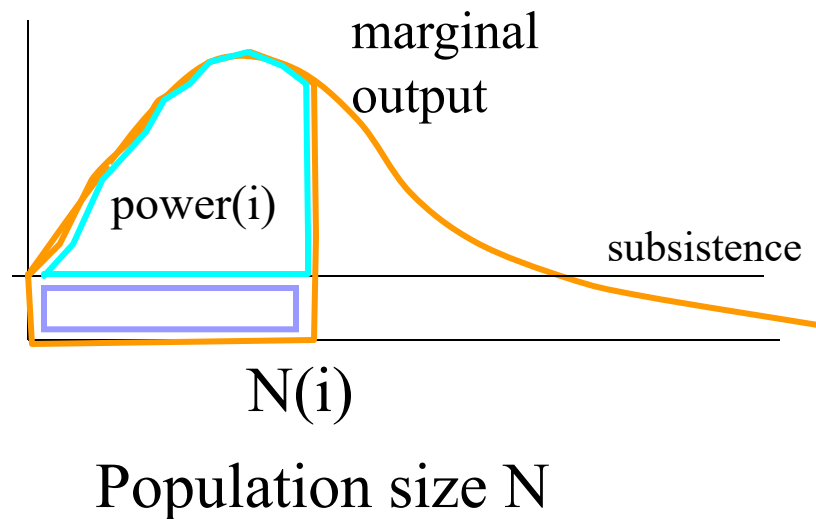
*What size is economic optimum (max of AP)?*

# Power optimum (issues)

- Let's not think about average individual but about the sum of the society
- Can think of a ruler, or of entire society (*"La France"*)
- Sauvy defines **power as the sum of production beyond subsistence** (for pyramids, armies, culture, ...)
- Question: *Do more people always mean more power?*

# Marginal productivity

- $MP(i)$  = output of the  $i$ 'th additional person (the additional increment of production) (*NOT the average*)
- $power(N) =$   
 $\text{sum}(\text{marginal output of first } N \text{ people}) -$   
 $\text{their subsistence requirements}$





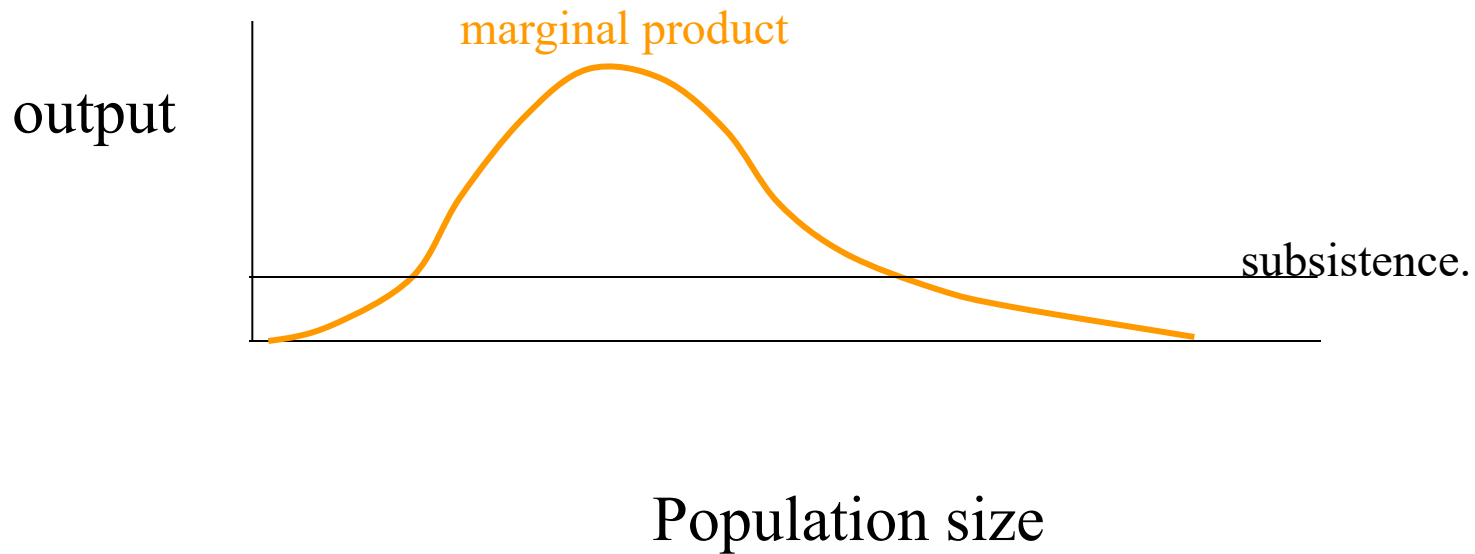
## Example (cont.)

N	1	2	3	4	5	6
MP	0	4	3	2	1	0
AP	1	2	2.33	2.25	2	1.67
Subsist.	2	2	2	2	2	2
"Power"	-1	0				

*What size maximizes power?  $\text{sum}(MP - S)$*

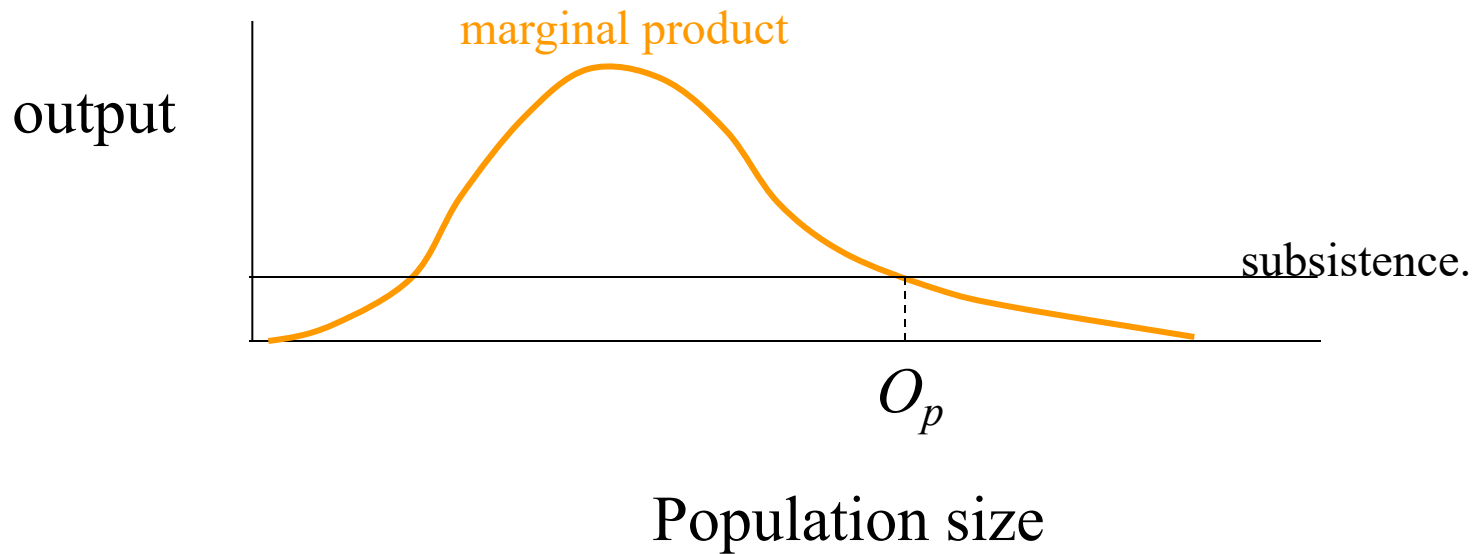
# The Power Optimum

What population size maximizes power?



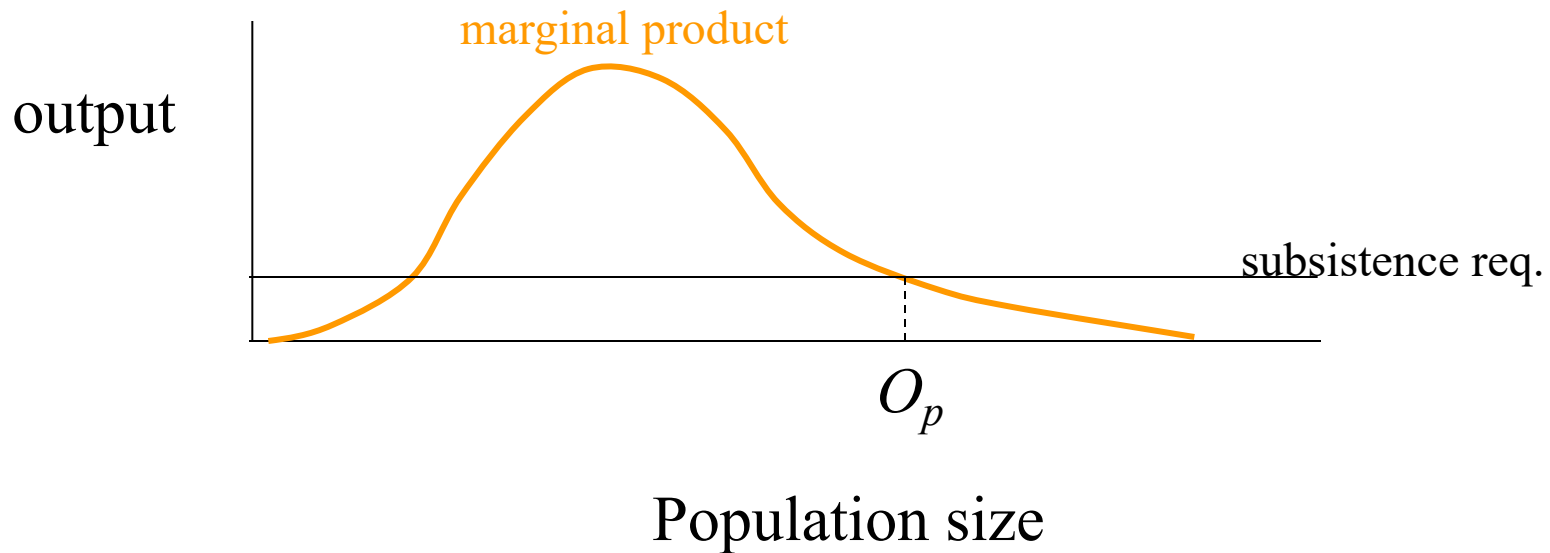
# The Power Optimum

What population size maximizes power?



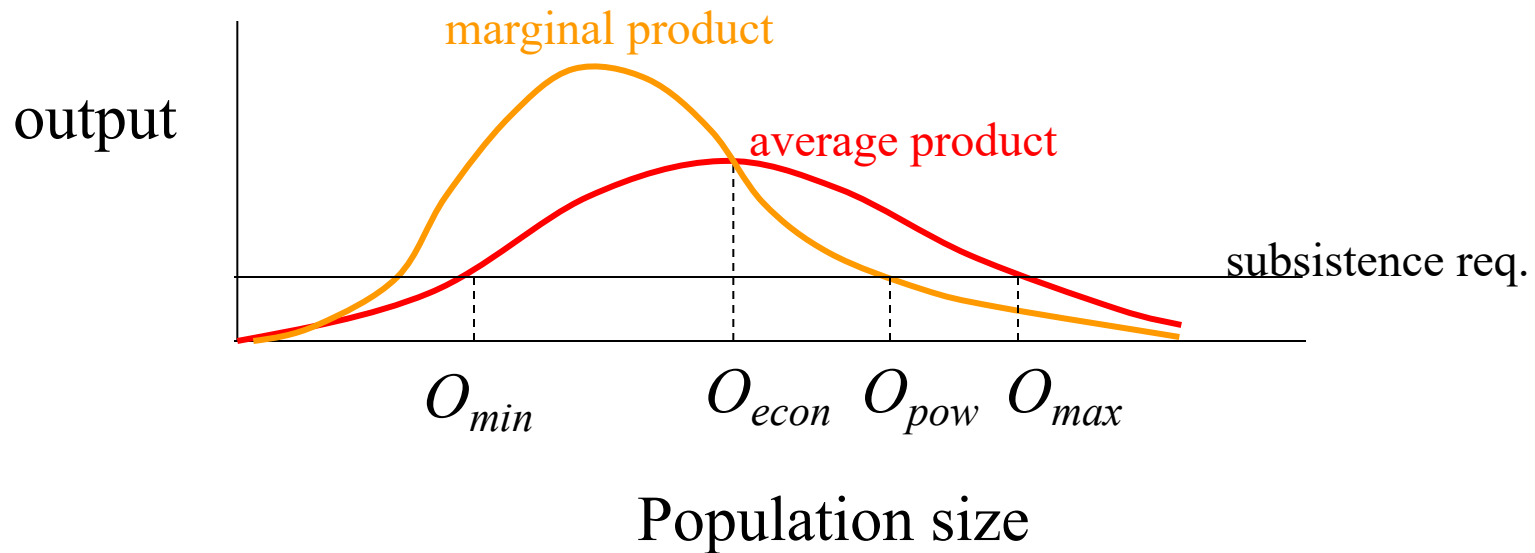
# Why does equality with subsistence mark the power maximum?

- Below  $O_p$ , each additional person increases social surplus
- Above  $O_p$ , each person is a “net cost”



# Ordering the optima

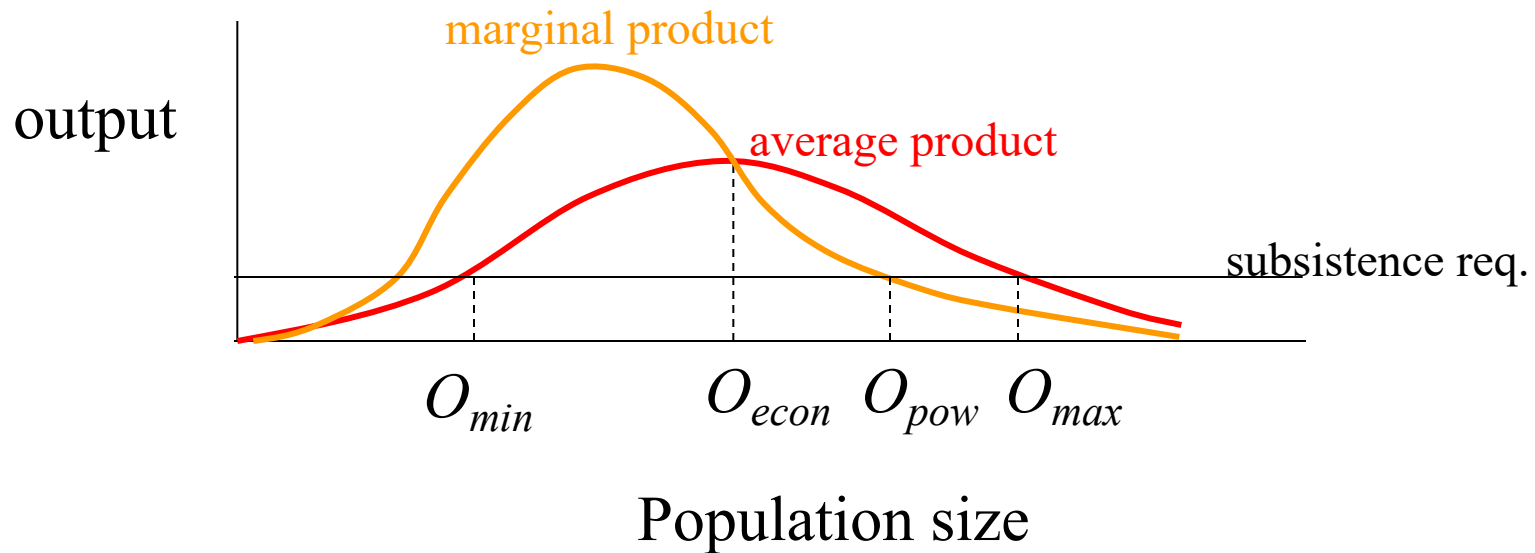
*Econ. optimum < Power optimum < Max. population*



- Q1. Why does intersection of MP and AP occur at max(AP)?
- Q2. How can people survive after  $MP < \text{subsistence}$ ?

# Ordering the optima

*Econ. optimum < Power optimum < Max. population*



Q: Does this ordering always hold? (With any MP pattern?)

# Population “evolution”?

If technology and other factors remain constant -- a big “if” -- then a growing population will experience the following stages:

1. The minimum (viable) population
2. Maximum marginal productivity
3. Maximum standard of living (econ opt)
4. Maximum power (power opt)
5. Maximum (viable) population

# Government and choice of optimum

“Before the 19th century this desire for power used to be so predominant that it hampered for a long time the understanding of the economic optimum. At about the time when this was understood, regal authority began to decline.”

(Sauvy p. 62)



# Conclusions

- Theoretical optimum, not a requirement (doesn't mean we should kill off our pop if we are over the optimum)
- Ignores feedback between pop size and technological level
- BUT, still a very useful framework:
  - the relative positions of different optima in general
  - we can understand how optimum depends on objective (whose perspective)

# Total Product: Understanding Sauvey's Figure 14

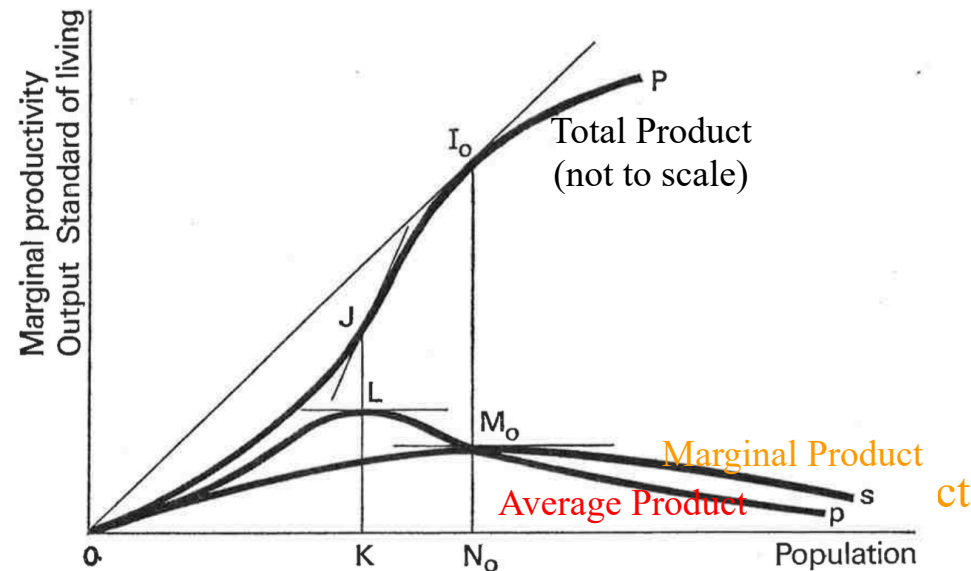


Figure 14. Marginal productivity, total output and standard of living.

- *Total prod = sum (MP)*  
$$TP(N) = \sum_{i=1}^N MP(i)$$
- *Tangent TP = MP*  
$$TP' = MP$$
- *Angle from origin to TP is AP. Why?*

TP picture will be useful when working with Solow model  
MP picture will be useful with Malthus, Migration, and elsewhere