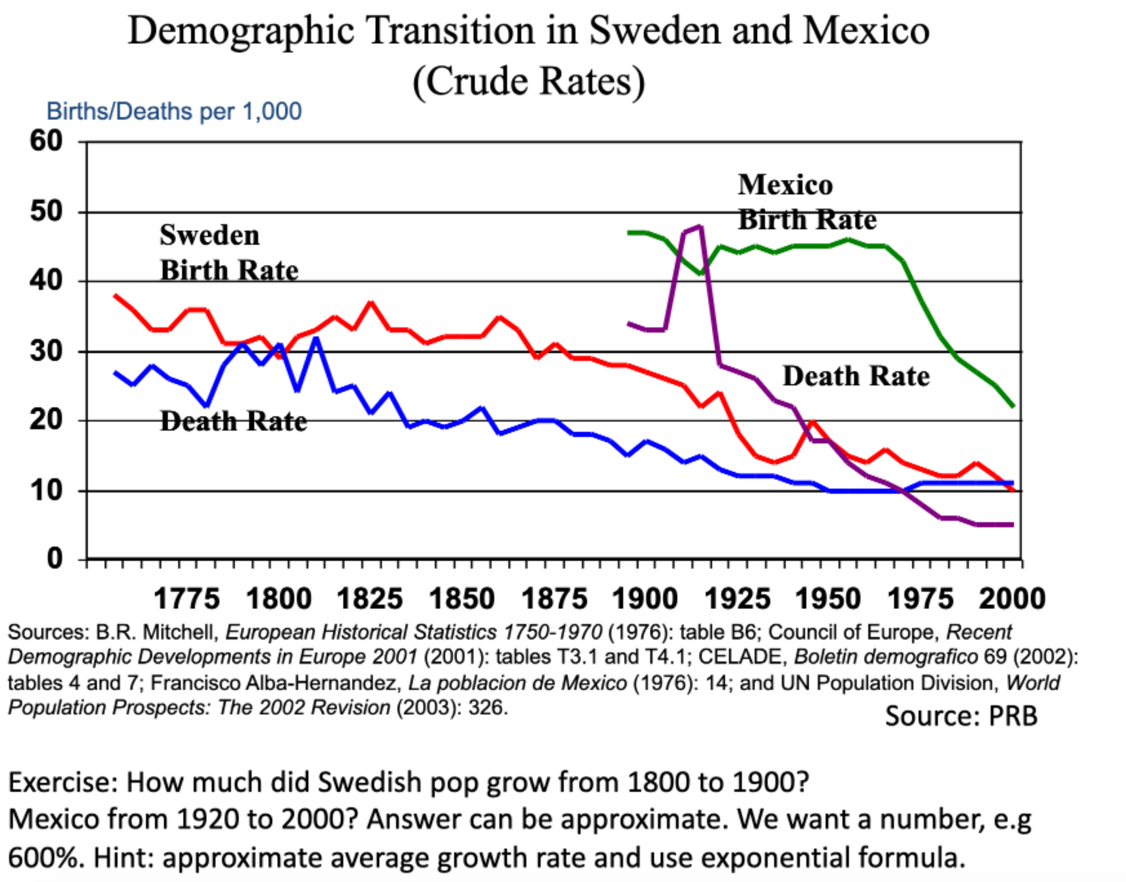
Calculating Exponential Growth in Sweden and Mexico
Supplemental Handout for Lecture 2

January 20, 2023

In Lecture 2, we saw this question:



How do we approach it?
Answer: Remember our exponential growth rate formula

$$N(t) = N(0)e^{Rt}$$

We are interested in the increase, i.e. the ratio $\frac{N(t)}{N(0)}$. So we want to calculate e^{Rt}.

For Sweden, t is 100 and for Mexico, the same number is 80. However, R, the difference between crude birth rate b and crude death rate d, fluctuates during those time periods. There are thus two ways of calculating our desired ratio, either by using an average value of R over the period (this is what we did in class through a simple visual inspection of the graph).

The other way to do this is to realize that in e^{Rt}, Rt can be broken up into the sum of individual units of t (years) and the value of R associated with them. We could write $e^{R_{1800}+R_{1801}+\ldots+R_{1900}}$. Or if we knew that R was unchanging in years 1800 and 1801, we could write $e^{R_{1800}\times 2+\ldots+R_{1900}}$.

It is difficult to precisely calculate the differences between b and d from this figure, but we can get a rough sense by visual inspection: from around 1810 to 1900, Sweden saw an R of around $10/1000 = .01$, and from 1800 to 1810, it saw around $3/1000 = .003$ (there are some fluctuations between years, but this is not likely to make a big impact in this calculation). Using these numbers:

$$e^{.003*10+.01*90} = e^{0.93} \approx 2.5$$

This is of the same order of magnitude as the in-class number of a 2-fold increase.

Getting numbers from a visual inspection of the Mexico data is a bit harder, but we can get a rough idea: .016 from 1920-1935, .022 from 1935-1940, steadily increasing but with an average value of around .03 from 1940-1975, .034 from 1975-1980, and .015 from 1980-2000. This gives us:

$$e^{.016*15+.022*5+.03*35+.034*5+.015*20} = e^{1.87} \approx 6.5$$

This is of the same order of magnitude as the in-class number of a 7-fold increase

If interested, you can look at the graph more carefully and calculate a more precise estimate (or look for the source data online), but this is the general approach you would use.

A key point to note here is that this is an approximation—if we wanted a more exact number we would use integrals. But this approach can give us a relatively good estimate, and help us understand differences in the way in which demographic transition was experienced in these two countries, with Mexico growing faster in a shorter period of time as a result of a much higher R that outweighed any differences in t.