Today’s agenda

• Solow cont.
  – Technology
  – Income Shares

• Piketty and Inequality
  – Cobb-Douglas and Beyond
  – Quantitative impact?
Solow and Technology Improvement

- Let’s include a factor “A” for technology

Y = A f(K, L)

Constant returns to scale →

- y = Y/L
  = (A/L) f(K,L)
  = A f(K/L, L/L)
  = A f(k, 1)

- So y increases with A
Is technology effect permanent?

- Do we stay at $k^*$?

- Why don’t we slip back, like Malthus?

- Solow ratchet vs. Malthus gerbil in a wheel

$$y = Af(k)$$

Output per worker per year ($y$)

$$(n+d)k$$

Savings

$savings'$

Capital per worker, $k$

$y' = A'f(k)$

$y^*$

$k^*$

$k^*$'
Technology effects: a two-step

First, we have effect of higher productivity with original amount of capital
\[ y(k^*) \rightarrow y'(k^*) \]

Second, we have effect of capital deepening
\[ k^* \rightarrow k^*’ \]

[Can see in app – in Lab]
Solow technology “ratchets”

- Each improvement gives us a new equilibrium, not just temporary
- Different from Malthus
  - Population ate away any improvement
  - Need more $k$ but can support it with higher $y$

Per capita output $y$ increases at same rate as technology $A$ improves
So what does Solow explain?

• Without tech change, neo-classical growth gives us a way for population to grow without hurting income.

• Population grows at some rate; Economy grows at same rate; per capita output constant
So what does Solow explain?

• Without tech change, neo-classical growth gives us a way for population to grow without hurting income.

  → Population grows at rate $n$; Economy grows at rate $n$; per capita output constant

• (This is the answer to the GREEN iclicker question we had on Tuesday)
So what does Solow explain? (cont.)

• *With* tech change, neo-classical growth gives us a way for population to grow and income to grow

• Say $A(t) = A_0 e^{gt}$

• → Population $N$ grows at rate $n$;

• → Economy $Y$ grows at rate $n + g$;

• → per capita output $y$ grows at rate $g$
What doesn’t Solow explain?

Exogenous factors:

• Technology

• Population

• (Also, savings rate $s$)
Solow Conclusions

- Solow approach retells the Malthusian story: a different steady-state
- Good news:
  - Can accommodate constant population growth without worsening wages (not possible in Malthus)
  - Technological change creates permanent improvement (not transitory like Malthus)
- Bad news:
  - Faster population growth implies lower income (unless forego consumption and keep savings up)
  - Key to long-term per capita growth is technology, not savings.
Growth and Inequality

Piketty’s argument
Stylized fact #1: Inequality’s fall and rise
Stylized fact #2
Growth’s rise and fall

Fig. 4. Rate of return versus growth rate at the global level, from Antiquity until 2100. The
Piketty’s capital idea

• Maybe mechanism is that lower growth increases capital per worker $k$ (via Solow effect)

• And maybe more capital per worker increases income inequality? (How could this be?)
Piketty’s argument

1. Slower growth $\rightarrow$ more capital per person (The neo-classical result)
2. More capital per person increases capital share of the economy (next)
3. Capital income more unequally distributed than labor income (right away)

QED: lower growth increases income inequality
Piketty 3. Income from capital is much more unequal than labor income

Table 1: Piketty’s estimates of labor and asset income received by the top decile for various inequality regimes with our estimate of the effect on total income of a unit increase in the capital/income ratio $\beta$

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Labor income ($H_l$)</td>
<td>20%</td>
<td>25%</td>
<td>35%</td>
<td>45%</td>
</tr>
<tr>
<td>Asset income ($H_k$)</td>
<td>50%</td>
<td>60%</td>
<td>70%</td>
<td>90%</td>
</tr>
<tr>
<td>Total income ($H_{l+k}$)</td>
<td>25%</td>
<td>35%</td>
<td>50%</td>
<td>60%</td>
</tr>
<tr>
<td>Effect of $\beta$ increase ($dH_{l+k}/d\beta$)</td>
<td>1.50%</td>
<td>1.75%</td>
<td>1.75%</td>
<td>2.25%</td>
</tr>
</tbody>
</table>

Note: For example, if $\beta$ were to increase by 2.0 from a ‘low inequality’ baseline, then the top decile share of income ($H_{l+k}$) would increase from 25% to 28% ($2.0 \times 1.50\%$). The first three lines of this table are from Piketty (p. 247-249). The derivative is our calculation based on change in weighted average of top decile share of labor earnings and capital earnings, assuming new capital earnings are perfectly correlated with existing capital earnings.

Source: Goldstein & Lee (2014)
(Back to Piketty 2)

Some accounting

- $Y = Y_l + Y_k$

- Assuming perfectly competitive markets
  $Y_k = MP(K) * K$
  and
  $y_k = MP(k) * k$

- What are $Y_l$ and $y_l$?
Marginal product

• Answers the question: if we increase an input factor, how much does output increase
• The slope of the production function (a.k.a. the derivative)
The marginal product of (k)apital = the slope of the production function

To do:
Sketch how MP(k) changes with k.

Does it go up, down, stay constant?

Does this remind you of anything in Malthus?
Distribution of income

In competitive economy, capital and labor each receives its marginal product:

Wage per person = mp(L)
Return on capital = mp(K) = f'(k)

Per capita output: \( y = f(k) \).
Of this, return on capital = \( k \times mp(K) = k \times f'(k) \)
So, wages = \( f(k) - k \times f'(k) \)

If population growth falls, output increases, wages increase, and return on capital falls. (cf. Piketty)
Does capital intensification increase capital’s share of income?

• Yes, because there’s more capital
• No, because rate of return on capital goes down
• Which effect is stronger?
• (Answer: it depends on how quickly MP declines)
Share of income from capital

$$\text{Share}_{\text{capital}} = \frac{y_k}{y} = \frac{\text{MP}(y) * k}{y}$$
Cobb-Douglas: capital intensification cancels out

• With Cobb-Douglas: \( y = k^a \)

• We calculate MP
  \[ MP(k) = \frac{dy}{dk} = a \cdot k^{a-1} \]

• We then substitute into
  \[ \text{Share}_{\text{capital}} = MP(k) \cdot \frac{k}{y} \]
  \[ = (a \cdot k^{a-1}) \cdot \frac{k}{k^a} = a \]

• So capital intensification exactly balanced by diminishing marginal returns. Share of national income from capital a constant, \( a \)
2. But what if MP(k) declines more slowly?

- Can still have diminishing marginal returns
- But now increase in capital won’t be fully offset by declines in MP(k)
- **Result is increasing share of national income goes to capital owners.**
- This is what Piketty highlights as possible.
- (Automation and robotization have slow declining MP, he says)
Piketty’s argument

1. Slower growth $\rightarrow$ more capital per person
   (The neo-classical result)
2. More capital per person increases capital share of the economy
3. Capital income more unequally distributed than labor income

QED: lower growth increases income inequality
Dramatic reading?

• Piketty and Saez p. 841
Population growth and inequality

- Slower population growth deepens capital stock (Solow)
- Per capita output will be higher.
- But Piketty argues that capitalists will be bigger winners than workers → more inequality
Next week

• Tues: Understanding technological change: intensification or innovation?

• Thurs: Are we doomed?
  (Running out of resources)